A Novice Teacher's Conception of the Crucial Determinants of his Teaching of Beginning Algebra

Cecilia Agudelo-Valderrama
Monash University
<Cecilia.Agudelo@Education.monash.edu.au>

This paper arises from a study whose overall purpose was to investigate the relationship between mathematics teachers' conceptions of beginning algebra and their conceptions of their own teaching practices. Drawing from a larger corpus of data collected over a 6-month period, the paper examines the case of a novice teacher's conceptions of his teaching of beginning algebra, highlighting the tensions afforded by his conceptions of the contextual factors of teaching. The data show that although this teacher greatly emphasised that his knowledge and dispositions were the crucial determinants of his teaching, in the light of his increased knowledge of the contextual factors of teaching, restructuring his teaching meant to him that these contextual factors were also to be considered crucial as he had to fulfil the requirements of his teaching job.

> If I were totally free I would be able to dedicate more time in the classroom to work in that sort of activity, but one is always behind with the program, so I have to hurry up! … due to the *time* factor, I sometimes have to do things like Teacher A (i.e., telling). ... If I were going to teach in Grade 8 next year, now that I know how this school works, I will have to do activities that are much shorter because time is short and you have to cover the program. (Pablo, Sep. /02)

This paper focuses on a Colombian novice teacher's conceptions of his own teaching of Grade 8-algebra. The vignette above provides an opening into Pablo's conceptions of the determinant factors of his teaching practice. The General Law of Education issued in 1994, and currently in force, calls for the need for flexibility in school curricula, urging Pablo as a teacher, to participate actively in the construction of his school curriculum, as education is to be improved by attending to the needs of the specific communities which schools serve (Article 78). Curriculum guidelines emphasise a shift in teaching methods from the traditional "chalk and talk" to a "hands-on" approach with more active participation on the part of the students (Díaz, Solarte & Arce, 1997).

Before the issuing of the General Law of Education, the National Curriculum for mathematics education was to take place according to a prescriptive "Pure mathematics model" (Robitaille & Dirks, 1982), based on the formalistic aspect of a hierarchically organised list of topics. Consequently, algebra was a packaged course to be taught in Grades 8 and 9. Curriculum statements were translated into pupils' textbooks that became the sole focus of teaching and learning all round the country (Mockus, 1985), and this type of textbook continues to be the only curricular materials available for teachers.

All secondary school leavers need to sit the National External Examination, where standardised tasks are used in order to control admission to higher education. The examination is also used as an indicator of the academic quality of the schools.

Research context

Much quantitative research about mathematics teachers' "conceptions" and "beliefs" has informed us about the degree of consistency between teachers' beliefs about mathematics and their beliefs about mathematics teaching. However, studying teachers' beliefs about mathematics and its teaching out of the actual context of teachers' classroom practice does not tell us much about the difficulties of teacher change. The qualitative studies, in which mathematics teachers' actual practices have been researched, have also

focused attention on identifying the consistency or otherwise between teachers' professed beliefs and their actual practice, suggesting that a teacher's beliefs about mathematics is the main factor responsible for a teacher's teaching practice (see for example, Gregg, 1995; Raymond 1997). Gates (2001) contends that the position that it is the teacher's view of mathematics that is responsible for classroom practice—which has driven much research on teachers' beliefs— needs to be questioned and deconstructed. Arguing that "without further clarification of these [research findings], one reading is that one's conception of mathematics is the deciding factor in structuring one's teaching" (p. 20), he further challenges the conclusion that a teacher's view of mathematics is the main cause for the stability of traditional practice in the mathematics classroom.

I would like to argue that in order to gain some understanding of the phenomenon of the stability of mathematics teaching practices in the Colombian context we need to pay attention to teachers' practical arguments (Richardson-Koehler & Fenstermacher, 1988) in their specific teaching situation, and to the complexity of teachers' practical knowledge. We need to study not just the teachers' conceptions of mathematics and mathematics teaching but also their conceptions of their own teaching practice. In other words, we need to focus on what teachers see as the deciding factors when structuring their own teaching practice, and on *why* and *how* those factors impact on their conceptions of their teaching practice. Do teachers see their conceptions of mathematics as the decisive factor or crucial determinant of their teaching practice —as researchers see it?

The overall aim of the study from which this paper arises was to investigate the relationship between mathematics teachers' conceptions of beginning algebra and their conceptions of their own teaching practices. When studying the teachers' conceptions of their own teaching of beginning algebra, which in the Colombian context takes place at Grade 8, they emphasised *the crucial determinants of their teaching*, hence the focus of this paper. The term *conceptions* in this study has been defined to encompass teachers' knowledge, beliefs and attitudes (see Agudelo-Valderrama & Bishop, 2003), as it has been shown that beliefs and attitudes as affective components of teachers' thought structures have a powerful impact on teachers' make up and approach (Ernest, 1989).

Methodology

The data collection—which took place over a six-month period, April to Sep 2002—was divided into two phases. In Phase 1, whose aim was to identify a variety of conceptions from an initial group of teachers in order to select case studies, data were collected from a group of 13 mathematics teachers, who taught at six different (state and private) schools in Bogotá. The participating teachers varied in ages and teaching experience and were teaching in Grade 8 during 2002. In Phase 2, a multi-case study with the participation of nine selected teachers was carried out.

Pablo was chosen as the focus of this paper because of the striking difference between his conceptions of his teaching of Grade 8 algebra and those of the rest of the participating teachers. Although all teachers followed a syllabus with the same list of topics, Pablo's teaching was guided by a model of learning in which he felt responsible for the organisation of "activities that helped all pupils construct the concepts" and the rules of operation, as opposed to "just telling" and following the structure of the textbook.

Data Collection and Analysis

A number of research strategies were used during data collection as shown in Table 1.

Table 1
Data collection strategies

Phase 1		***Phase 2***	
Research instrument/activity	Purpose	Research instrument/activity	Purpose
Questionnaire 1	Collect data about the teachers' conceptions of *the why*, *the what* and *the how* of the teaching of G 8-algebra	Observation of five consecutive lessons	Collect data to further understand the teachers' portrayed practices
Interview 1 (Questionnaire 1 follow-up interview)	Explore the teachers' reasons for answers in Questionnaire 1	Interview 2 (Observation follow-up interview plus Concept map activity)	Obtain teachers' explanations of classroom incidents & engage them in the construction of a concept map of the determinants of their teaching of Grade 8-algebra
Questionnaire 2	Identify the teachers' interest in a problem-based teaching approach	Focus Group	Identify the teachers' concerns and explore further their conceptions of their teaching of beginning algebra
		Interview 3	Probe key ideas, not properly explored during the Focus Group & revise concept map

Data analysis was conducted in the language (Spanish) of the data collection. Data collected through the different sources was reviewed and classified in order to identify i) the teachers' conceptions of beginning algebra, and ii) the teachers' conceptions of their own teaching practices. In identifying the teachers' conceptions of beginning algebra, the focus of the analysis was placed on data related to the fundamental components of teaching (i.e., answers to the *why*, *what* and *how* of the teaching of beginning algebra). In identifying the teachers' conceptions of their own teaching, the focus was placed, at one point, on *why the teachers taught Grade 8-algebra in the way they did* and, at another point, on *why they would (or would not) be willing to consider a different approach in their teaching of Grade 8-algebra*.

Pablo's Case

Pablo was teaching in a private school that caters for children of a middle socioeconomic background, and he was in his second year of teaching. The school is a day school, and Pablo was "completely dedicated" to his job in that school. Pablo did his teaching practice for his Bachelor of Education degree (in mathematics) during 3 semesters, in grades 6, 8 and 9. He had taught for a year (in 2001) in grades 5, 6 and 7. Consequently, the year 2002 when he participated in this study, was his first time as a teacher of Grade 8. Pablo was very enthusiastic about participating in the study and spoke confidently about his teaching during the interviews. He also seemed to be confident in his teaching during the lessons observed.

Pablo's Conceptions of Beginning Algebra

For Pablo, "algebraic knowledge is important knowledge for understanding real life situations". The main reason why "pupils need to learn algebra is because it is knowledge useful for problem solving". Pablo believed that algebraic thinking could be promoted in the primary school, for example, helping children to make generalisations about their working methods.

> When children buy sweets, they can be encouraged to think about the formula for calculating the cost of any number of sweets, having the price of a sweet; that is without mentioning the word formula, but just saying it as a primitive algebra (writes and says, 'number of sweets × price of one sweet = cost').

Pablo emphasised the need to help pupils find connections between their already learnt mathematical concepts with the work with algebraic expressions and with pupils' everyday activities. He argued that, "If when studying mathematics pupils don't find connections with other mathematical concepts and with the world, then it has no meaning for them. One finds no reason to study it". His central aim for the teaching of Grade 8 of "helping pupils see the functionality of what they are learning" could not be achieved by following the textbook approach because

> textbooks bring just a list of exercises after a definition, or after an algorithm has been given. If one portrays mathematics as just a list of exercises that you have to do 'this way' because that's the way the textbook does it, without seeing something of where things could come from, then pupils don't see the point. ... To give the pupils a list of exercises telling them 'follow this rule' is to teach mathematics as dead mathematics.

Instead of initiating algebraic work with the presentation of a definition of algebraic expression, followed by a list of examples, Pablo wanted his pupils "to construct algebraic expressions", drawing on the concept of perimeter and area of rectangles, "because they needed to see where the expressions could come from, and why they need, for example, to add polynomials".

> I don't pay attention to the definitions that come in the textbook, that an algebraic expression is a combination of numbers letters and operations! When we started to talk about letters or simple expressions we did it because we were talking about area and perimeter of rectangles or triangles. Later, when we needed expressions with different letters, what I did was to draw on an activity that they were doing in PE with long jump where they were allowed to do 13 running steps for speed, and then the jump. We started talking about the distances that Martha and Juan would have travelled during their 13 running steps. I asked for the cases of other pupils who may have had longer steps. They concluded that even if each pupil ran 13 steps, the distance travelled was different for each pupil, and that the same applied for the distance travelled in the jump…

Although Pablo anticipated that the pupils would encounter, later, situations where they would use letters to represent specific unknowns, his intention was that the pupils used the letters, initially, to represent the general case. When explaining how he thought pupils would start seeing the letters as specific unknowns, he said:

> I want them to first see that the letters there [in the tasks with rectangles shown in next paragraphs] are representing any number, according to the particular situation we are talking about. Of course they will work later with the rectangles, where the letter is a specific unknown ... all the textbook exercises are about that, and actually they have done a bit of that in Grade 7.

As will be seen in the following paragraphs, although Pablo did not make explicit that when working towards the construction of the generalisations, he was trying to help pupils become familiar with the concept of variable to express relationships between quantities, the tasks with areas and perimeters he devised were intended to promote this. Examples of

the "simple tasks designed to help pupils construct algebraic expressions with the same letter" are shown in Figure 1.

What Pablo did with *Situation 1,* of Figure 1, was explained thoroughly by him at Interview 1. *Situation 2* has been taken from one of the observed lessons which took place "one week after the lesson when Situation 1 was worked out with the class". In both situations, after the figures were drawn on the board, pupils were asked to draw them in their books and then asked for the measure of the sides of their drawings [the measure expressed by using the small squares of their squared paper], because he "wanted them to see that *a* can represent a different number for each pupil, and that the areas of the figures depend on the value of *a*.

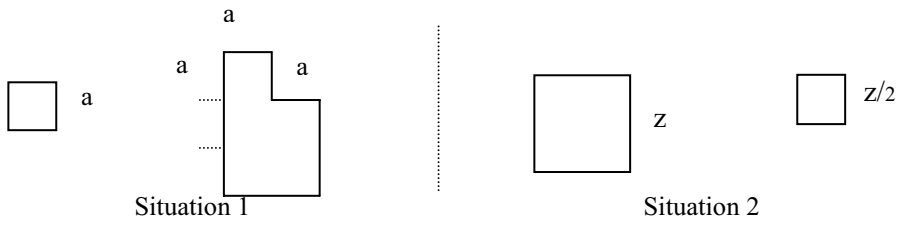

Figure 1. Pablo's "simple tasks" for introducing pupils to algebraic expressions.

In situation 2, however, apart from finding the perimeter and the area of the two given figures, pupils were asked to add the two areas. Pupils worked in pairs and sometimes discussed their ideas with other groups showing great enthusiasm about their work. Most pupils completed the task successfully. When Pablo brought the class together, and pupils were invited to show their work "to explain the method of calculation used", three pupils said that they had difficulties adding fractions and others had difficulty identifying the coefficient of Z^2 [when adding Z^2 to $Z^2/4$]. When the teacher asked them to think of equivalent fractions, two pupils suggested that that was too long and more difficult than cross-multiplying, according to the rule. Pablo asked what the rule was. A pupil went to the board and gave an example explaining the rule to add fractions with different denominator. The two pupils who were asking how to add fractions also said that the rule was better. Without any more comments about this, Pablo continued with the lesson.

These specific incidents were explained by Pablo at Interview 2. In relation to the difficulties a few pupils had shown identifying the coefficient of Z^2, he said:

> I am aware that some pupils don't understand. I have to design better activities but it is difficult to design activities that work for all of them. ... My main concern has been to give some more work to the fastest pupils because they get bored if they have finished and I carry on working with the ones that haven't understood. Sometimes I have had to improvise with the fastest because they start disrupting if they have nothing to do. I design the activities on the basis of what connects for me, but the important thing is that it connects for them.

In relation to some pupils preferring to be given rules for adding fractions, Pablo explained that some pupils like to learn rules by heart.

> They want just to be given simple rules. I have to rush because I am behind with the program and many pupils are anxious because they know we are behind. They think that we haven't started algebra because I am not teaching the ten cases of factorisation that their friends from other schools already know!

Promoting connections between concepts as Pablo's first teaching style priority. Pablo explained at Interview 1 that, in Questionnaire 1, he had ranked 'Designing classroom

work that promotes connections between different mathematical topics studied' as his number-one priority for both preferred and actual practice because that is what he was trying to do in his teaching. He emphasised the word 'trying' here, because he was "just starting to see" how his ideas for teaching worked. "I am trying; I am trying because I am just starting to learn how to organise activities. I am trying to see and learn how they [the pupils] do their work so that I can guide myself when thinking of the activities". He pointed out, at Interview 2, however, that he was feeling pressure, "first of all, because the time available for teaching was very short as the school was always organising extracurricular activities without taking into account the set timetable; and, secondly, because of what some pupils expected him to do in his teaching; "for example, the ten cases of factorization".

Learning to factorise according to the ten cases presented in some popular textbooks was not learning for Pablo. At Interview 3, he argued that learning to apply the rule for each case was just manipulating specific expressions.

> If I really learn, for example, how to factorise $a^2 - b^2$, and why I factorise it in a specific way, I should be able to factorise the other types of polynomials or the other cases. And why do I have to teach factorisation as a separate topic if factorising is a way to know if I understand multiplication? If when knowing the sides of the figure they can find the area, then given the area, we find the sides. Pupils need to be asking themselves 'why am I doing this?' For me mathematics is analysis, and I don't have to be explaining much because, as I have seen it, pupils answer each others' questions.

Assessing pupils' work. Pablo had "two preferred forms of assessing his pupils (as he wrote in his responses to Questionnaire 1, and explained at Interview 1): "i) monitoring continuously the pupils' work, which [he did] by collecting the pupils' notebooks, to see what they [had] done during the lessons and, obviously, in their homework … and ii) through written tests, where pupils can apply what they have learnt during a period of time, without partitioning the topics". But this could not be managed as he "had to do tests after each topic". Pablo explained that he was aware that tests were "not the best way to identify pupils' difficulties ... you see them working very well in class and yet they make mistakes in the tests". So why did Pablo give tests at all? He had "to give a test after each topic" was taught

> because of the deadlines to hand in the pupils' grades; and following each pupil in a personalised fashion is just not possible; so I have to do something that can be manageable for everybody, in a simple way. … I have to hand in, to the Academic Coordinator, the grades in relation to the content objectives stated in the Grade 8 program for that specific term. … Those objectives represent the 'assessment indicators' for which I have to provide pupils' grades for the school assessment report.

Was Pablo supposed to question this? Curriculum statements and the school's assessment structure, the expectations of his pupils and their parents (not to mention the External Examination requirements) represented for him what he was accountable for.

Pablo's Conceptions of the Determinants of his Teaching Practice

Pablo maintained throughout the first five months of the data collection process that the crucial factors influencing his teaching were his knowledge and his dispositions as a teacher. By his dispositions as a teacher he meant, "something like the teacher's philosophy of the teaching of mathematics":

> (i) The time dedicated to prepare classroom work. For example, there may be some [teachers] who just repeat the same set of questions every year. (ii) The desire to improve what one does and the interest in increasing what one knows. There are some colleagues that want to stay in grades 6 and 7 all the time because 'Oh no! I don't want to teach in grades 10 or 11 because I haven't taught in

those grades for a long time'. (iii) The enjoyment of what one does. Do I do this because I have to, or because I want to? Some teachers don't want to be more than just the *repetitors* of a routine. In one word, it is the philosophy that one has about the teaching of mathematics.

As we saw in the previous section, he justified the fact that some pupils did not make the connections by the inadequacy of the tasks he had designed, and to his limited knowledge of teaching. Despite the fact that Pablo declared at Interviews 1 and 2 that he felt pressure from the pupils and their parents to cover the set program, and from the shortage of teaching time, within the list of factors that he provided when constructing his *initial* concept map of the determinants of his teaching, he identified his knowledge and dispositions as the *number-one* determinants (see Figure 2). However, as the end of school year approached, he started to emphasise "the great limitations that time represent[ed] for his teaching" (Focus Group session, September/02).

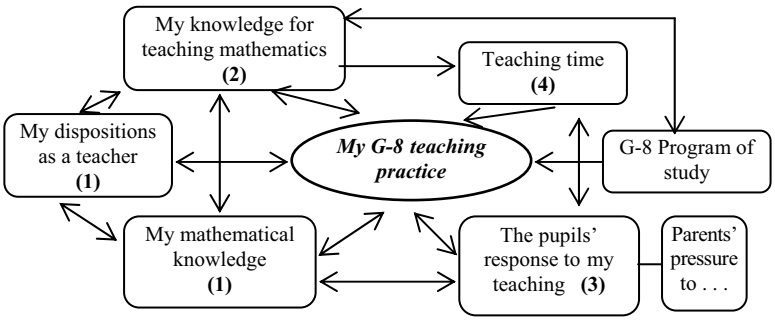

Figure 2. Pablo's initial concept map of the determinants of his teaching practice.

At Interview 3, he pointed that the activities discussed at the Focus Group were excellent, but that "those require[d] enough time to work with the pupils". He added, "If I were totally free I would be able to dedicate more time in the classroom to work in that sort of activity, but one is always behind with the program, so I have to hurry up!" When asked if there was something that he would change in his teaching of Grade 8, he said, "It isn't *change* because it will be the same approach. The adequate term would be *improvement* of the activities in order to help all pupils understand. But I don't know because due to the *time* factor*,* I sometimes have to do things like Teacher A (i.e., telling). ... If I were going to teach in Grade 8 next year, now that I know how this school works, I will have to do activities that are much shorter because time is short and you have to cover the program".

The identification of time as a crucial factor in his teaching was reconfirmed in his revision of the concept map of the determinants of his teaching practice. In his *final* concept map, where he was presented with the factors (i.e., the boxes) of the initial map, he drew the same connecting arrows between the factors, but when asked to identify the strength with which the factors affected his teaching, he gave a different number (from the one he had given in his initial map) to 'Teaching time'. He wrote the number "1" in the 'Teaching time', 'My dispositions' and 'My knowledge' boxes, and explained, once again, "Now that I know how this school works, I know that the use of time is my main priority".

Conclusion

Pablo's case provides an illuminating example for discussion, when identifying barriers and possibilities for mathematics teaching change in Colombia in relation to the broader questions of the large study. The process of change observed in Pablo's conceptions of the

crucial determinants of his teaching illustrates very well the differences we can observe between the researchers' and the teachers' explanations for the stability of traditional teaching approaches. Before Pablo had sufficient knowledge of the contextual factors of teaching, he had identified *internal* factors (his knowledge and dispositions) as the crucial determinants of his teaching, but once he gained more knowledge of the contextual factors, he could not ignore the pressures exerted by the *external* factors. Can we say that Pablo's conception of beginning algebra was the deciding factor in structuring his teaching? These findings suggest that teachers' practical knowledge is complex and that, as Gates (2001) has pointed out, teachers' knowledge for teaching includes teachers' social knowledge. Consequently, studying teachers' conceptions of their teaching of mathematics requires us to consider teachers' social knowledge, beliefs and attitudes as key dimensions, if we are to gain some understating of the barriers and possibilities of teacher change.

References

Agudelo-Valderrama, C., & Bishop, A. J. (2003). Teachers' conceptions of school algebra and its teaching: preliminary findings from a study in Colombia. In L. Bragg, C., Cambell, G., Herbert & J. Mousley (Eds.), *MERIN: Mathematics Education Research: Innovation, Networking*, *Opportunity* (Proceedings of the 26th annual conference of the Mathematics Education Research Group of Australasia, Vol. 1, pp. 49-56). Geelong: MERGA.

Díaz, C., Solarte, E., & Arce, J. (1997). Colombia. In Robitaille. D. F. (Ed.), *National contexts for mathematics and science education. An encyclopedia of the education systems participating in TIMSS* (pp. 82-90). Vancouver: Pacific Educational Press.

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teachers: a model. *Journal of Education for Teaching*, *15*(1), 13-33.

Gates, P. (2001). Mathematics teachers' beliefs systems: exploring the social foundations. In M.Y.D. Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Annual Conference of the International Group for the Psychology of Mathematics Education*, Vol 3, (pp. 17-24). Utrecht, The Netherlands.

Gregg, J. (1995). The tensions and contradictions of the school mathematics tradition. *Journal for Research in Mathematics Education, 26*(5), 442-446.

Mockus, A (1985). La reforma curricular y el magisterio. *Educación y Cultura*, Edición 4, 65-85.

Raymond, A. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education, 28*(5), 550-576.

Richardson-Koehler, V. & Fenstermacher, G.D. (1988). *The use of practical arguments in staff development*. Paper presented at the annual meeting of the American Association of Colleges for Teacher Education, New Orleans, LA. (ERIC Document Reproduction Service No. ED 291744)

Robitaille, D. & Dirks, M. (1982). Models for mathematics curriculum. *For the Learning of Mathematics*, *2*, 3-19.